PCMI Mirror Symmetry Lectures

Problem Set 1

1. This is a calculation with superfields that everyone should do once It takes some time but helps to understand how it works. Take the action for the nonlinear sigma model

$$\mathcal{L} = K(\Phi, \bar{\Phi})$$

for some real function $K(x, \bar{x})$. Perform the Berezin integrals to obtain

$$L = \int d^4\theta \, \mathcal{L} \ ,$$

using

$$\int d^4\theta = \frac{1}{4} \int d\theta^+ d\theta^- d\bar{\theta}^- d\bar{\theta}^+ \ .$$

Eliminate the auxiliary field to obtain the action we wrote in lecture.

2. This is a tough problem, but it is a good way to become familiar with the $N = 2$ superconformal algebra, and introduces an important concept, *spectral flow*, that I will not discuss in lecture. Recall that equation (2.8) has a parameter a which determines the moding of the fermionic currents. In fact, shifting a is an automorphism of the algebra. Show this by constructing the map from generators at $a = 0$ to generators at arbitrary a preserving (2.8). To start you off, consider the algebra satisfied by

$$\begin{aligned} L'_n &= L_n + (a - \tfrac{1}{2}) J_n + \frac{c}{6} (a - \tfrac{1}{2})^2 \delta_{n,0} \\ J'_n &= J_n + \frac{c}{3} (a - \tfrac{1}{2}) \delta_{n,0} \\ G'^{\pm}_r &= G^{\pm}_{r \pm a \mp \frac{1}{2}} \ . \end{aligned}$$

3. Show that (2.8) holds for the modes of the currents in (2.16), using the classical brackets for the free fields.

4. Most of you can probably do this in your sleep, so don't bother. But it is fun to show explicitly that the quotient of the quintic hypersurface in CP^4 has the right Euler number. The quotient can be described as follows. The hypersurface

$$x_1^5 + \cdots + x_5^5 = 0$$

is acted upon by a $\mathbb{Z}_5^4$ symmetry acting as $x_i \to \omega_i^r x_i$ where ω is a fifth root of unity and $0 \le r_i \le 4$. This looks like $\mathbb{Z}_5^5$ but one of these acts trivially. The mirror is the quotient by the subgroup of symmetries satisfying $\sum r_i = 0 \pmod 5$.