Physics 342

Corrections to the Notes - Useful for HW2

Since I messed up in computing Π' here is the correction Jie found and its consequences.

The notes are correct until the point at which I introduced a Feynman parameter and combined denominators to get

$$-i\Pi_f(k^2) = -4g^2 \int \frac{d^4p}{(2\pi)^4} \int_0^1 dx \, \frac{p(p+k) \pm m^2}{(p^2 + k^2x + 2xpk - m^2 + i\epsilon)^2} \ .$$

I now shift the integration variable from p to $q = p + xk$, in terms of which the denominator is spherically symmetric. The numerator is then

$$p(p+k) \pm m^2 = (q - xk)(q + (1-x)k) \pm m^2 \ .$$

When I expand this and drop all terms linear in q (assuming as usual that my integral is part of a convergent sum) I get now

$$-i\Pi_f(k^2) = -4g^2 \int_0^1 dx \int \frac{d^4q}{(2\pi)^4} \, \frac{q^2 \pm m^2 - k^2x(1-x)}{(q^2 + k^2x(1-x) - m^2 + i\epsilon)^2} \ .$$

I can now perform the integral using the techniques from class to get

$$\Pi_f(k^2) = \frac{g^2}{4\pi^2} \int_0^1 dx \, \left(\alpha m^2 - 3k^2x(1-x)\right) \log\left(m^2 - k^2x(1-x) - i\epsilon\right) \ ,$$

with

$$\alpha = \begin{cases} 1 & \Gamma = 1 \\ 3 & \Gamma = i\gamma_5 \ . \end{cases}$$

We can now use this to complete the computation of

$$\Pi'(k^2) = \Pi_f(k^2) - \Pi_f(\mu^2) - (k^2 - \mu^2) \frac{d\Pi_f}{dk^2}\Big|_{k^2=\mu^2} \ ,$$

which gives

$$\Pi'(k^2) = \frac{g^2}{4\pi^2} \int_0^1 dx \left\{ \left(\alpha m^2 - 3x(1-x)k^2\right) \log \frac{m^2 - k^2x(1-x) - i\epsilon}{m^2 - \mu^2x(1-x)} + x(1-x)(k^2 - \mu^2) \frac{\alpha m^2 - 3\mu^2x(1-x)}{m^2 - \mu^2x(1-x)} \right\} \ .$$

Using this correct expression should be helpful in getting the right answer.