Baryons in $dP_3$ Theories
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The $dP_3$ theories to which these notes apply are the $dP_3$ theory discussed in my thesis and the first $D_{12}$ symmetric $dP_3$ theory we found. The first thing to do is to write down the generators for the rings of baryonic operators on the moduli space of these theories, i.e. modulo classical equations of motion.

In the non-symmetric $dP_3$ theory, we have as generators

\[
\begin{align*}
&x_{42}^N, x_{21}^N, x_{26}^N, x_{13}^N, x_{63}^N, x_{35}^N, \\
&\left[(x_{13} x_{35})^{N-s} x_{15}^s \right], \\
&\left[(x_{63} x_{35})^{N-s} x_{65}^s \right], \\
&\left[(x_{42} x_{21})^{N-s} x_{41}^s \right], \\
&\left[(x_{42} x_{26})^{N-s} x_{46}^s \right], \\
&\left[x_{54} y_{54}^{N-s} \right], \\
&\left[(x_{21} x_{13})^{N-s} (x_{26} x_{63})^s \right], \\
&\left[(x_{34} x_{42})^{N-s} (x_{35} x_{52})^s \right], \\
&\left[(x_{35} x_{54})^s (x_{35} y_{54})^{t-s} x_{34}^{N-t} \right], \\
&\left[(x_{54} x_{42})^s (y_{54} x_{42})^{t-s} x_{52}^{N-t} \right].
\end{align*}
\] (1)

Let us now consider the baryons in the symmetric $dP_3$ theory. We can immediately write the baryonic generators:

\[
\begin{align*}
&x_{24}^N, x_{34}^N, x_{64}^N, x_{25}^N, x_{35}^N, x_{65}^N, \\
&\left[x_{12}^s y_{12}^{N-s} \right], \left[x_{13}^s y_{13}^{N-s} \right], \left[x_{16}^s y_{16}^{N-s} \right], \\
&\left[x_{41}^s y_{41}^{t-s} z_{41}^{N-t} \right], \left[x_{51}^s y_{51}^{t-s} z_{51}^{N-t} \right].
\end{align*}
\] (2)

Now I will use the fact that I know the mesonic invariants in both of these theories and have a strong geometrical interpretation of them. In particular, I know how to match (up to global symmetries) them from one theory to the other. I will use this information to learn how the baryons should correspond.
In the symmetric $dP_3$ theory, I have relations between baryonic and mesonic invariants of the form
\[
\begin{align*}
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_5^N, \ s = u = N \\
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_7^N, \ s = N - u = N \\
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_3^N, \ t - s = u = N \\
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_6^N, \ t - s = N - u = N \\
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_1^N, \ N - t = u = N \\
[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}] & \sim [x_{12}^u y_{12}^{N-u} x_{25}^N] \sim z_5^N, \ N - t = N - u = N,
\end{align*}
\]
and of course many more such relations. My notation here is somewhat sloppy. E.g. “$s = u = N$” indicates that the corresponding relation holds only for the indicated values of $s$ and $u$. The reason I choose to write the relationships as above, rather than perhaps the simpler $x_{12}^u y_{12}^{N-u} x_{25}^N \sim z_5^N$, is that we expect physically that baryons of the form $[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}]$ for all $s \leq t \leq N$ correspond to branes wrapping cycles in a given family on the horizon, and hence it is best to think of these baryons also together into one family, the relation holding only for particular extremal baryons in the family.

It is also clear that families of baryons like $[x_{51}^s y_{51}^{t-s} z_{51}^{N-t}]$ must correspond to similar families in the non-symmetric theory. We know that such families arise because of symmetries of the moduli space of 3-cycles on which the 3-branes are wrapping and this should also hold true in the non-symmetric theory$^1$.

So now we look for similar relations in the non-symmetric $dP_3$ theory, where by “similar relations” I mean relations between extremal members of some families of baryons, e.g. (3). So, for instance, corresponding to (3) above, we have in the non-symmetric $dP_3$ theory relations of the form:
\[
\begin{align*}
[(x_{35} x_{54})^s (x_{34})^{t-s} (x_{35} y_{54})^{N-t}] x_{42}^N & \sim [(x_{26} x_{63})^u (x_{21} x_{13})^{N-u}] \sim z_5^N, \ s = u = N \\
[(x_{35} x_{54})^s (x_{34})^{t-s} (x_{35} y_{54})^{N-t}] x_{42}^N & \sim [(x_{26} x_{63})^u (x_{21} x_{13})^{N-u}] \sim z_7^N, \ s = N - u = N,
\end{align*}
\]
e tc. There are many relations of this sort, and since they are of a special form, it is easy to determine which relations in one theory correspond to which relations in the other theory. From this, we can read off how the baryons themselves should correspond.

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$^1$ The relevant symmetries here are the continuous global symmetries which are identical in both theories, not the discrete symmetries.
Now, the strange thing that happens is that we almost do get a match of baryons in one ring to baryons in the other. From the sorts of relations I describe above, it is easy to see that the correspondence, if it is to work, should satisfy

\[
\begin{align*}
  x_{24}^N &\sim x_{35}^N \\
  x_{34}^N &\sim x_{26}^N \\
  x_{64}^N &\sim x_{21}^N \\
  x_{25}^N &\sim x_{42}^N \\
  x_{35}^N &\sim x_{13}^N \\
  x_{65}^N &\sim x_{63}^N \\
  [x_{12}^s y_{12}^{N-s}] &\sim [(x_{21} x_{13})^{N-s} (x_{26} x_{63})^{s}] \\
  [x_{13}^s y_{13}^{N-s}] &\sim [(x_{42} x_{21})^{s} x_{41}^{N-s}] \\
  [x_{13}^s y_{13}^{N-s}] &\sim [(x_{63} x_{35})^{N-s} x_{65}^{s}] \\
  [x_{16}^s y_{16}^{N-s}] &\sim [(x_{42} x_{26})^{N-s} x_{46}^{s}] \\
  [x_{16}^s y_{16}^{N-s}] &\sim [(x_{13} x_{35})^{s} x_{15}^{N-s}] \\
  [x_{51}^s y_{51}^{N-s}] &\sim [(x_{35} x_{54})^{s} (x_{34})^{t-s} (x_{35} y_{54})^{N-t}] \\
  [x_{41}^s y_{41}^{N-s}] &\sim [(x_{54} x_{42})^{s} (y_{54} x_{42})^{t-s} x_{52}^{N-t}] 
\end{align*}
\]  

(5)

But as is immediately obvious from (5), there is a problem. For relations in the symmetric theory in which, say, \([x_{13}^s y_{13}^{N-s}]\) appears, in the non-symmetric theory the corresponding relations occur with both \([(x_{42} x_{21})^{s} x_{41}^{N-s}]\) and \([(x_{63} x_{35})^{N-s} x_{65}^{s}]\). In relations in the symmetric theory involving \([x_{13}^s y_{13}^{N-s}]\) and one of the three baryons \(x_{25}^N, x_{35}^N, x_{65}^N\), we see the corresponding relations in the non-symmetric theory involving \([(x_{42} x_{21})^{s} x_{41}^{N-s}]\)—and this is indicated by the subscript 5. And conversely for the subscript 4.

So, this is why I say that I lack the relations among fields

\[
\begin{align*}
  x_{65} &\sim x_{42} x_{21} \\
  x_{41} &\sim x_{63} x_{65} \\
  x_{15} &\sim x_{42} x_{26} \\
  x_{46} &\sim x_{13} x_{35}
\end{align*}
\]  

(6)
and it is not clear that these sorts of relations make any sense at all anyway, because of course the states are charged under differing gauge-groups. I think it is very suggestive to think of the discrete symmetry breaking in this context, but I have little to say right now.

It is also true, by the way, that it still appears that I have some extra generators listed in the non-symmetric theory that I have not accounted for in the symmetric theory—although I have a strong suspicion that they occur in the symmetric theory as products of generators therein.