Physics 55
Homework Set 4

Due 2/8/11

1. The Moon, we are told, orbits Earth at a distance of 384,000 km with a sidereal period of 27.3 days. Assume a circular orbit.
   (a) What is its acceleration towards Earth? What causes this?
   (b) Use Newton’s law of gravity and the result of (a) to compute the mass of the Earth.
   (c) Saturn’s moon Janus orbits the planet at a distance of 151,472 km, with a period of 0.695 days. Use this data to compute the mass of Saturn. Note that we have computed the mass of the planet from data we can obtain by observing from Earth.

2. Manmade satellites obey the same Kepler’s laws as natural ones, of course, since the universal law of gravity applies to everything.
   (a) Satellites in low-Earth orbits are placed at altitudes of between 160 and 2000 km above the Earth’s surface. For a typical satellite orbiting at an altitude of 1000 km find the sidereal period of its orbit (Earth’s radius is 6400 km). At what angular velocity (in degrees per minute) does the satellite move about its orbit? When we observe low-Earth satellites, they will look like “stars” moving at this characteristic rate across the sky. We observe these only early in the evening soon after sunset (or, if we rise early, just before sunrise) but never near midnight. Why is this?
   (b) Communication satellites are not typically placed in low-Earth orbit since this would require satellite dishes, for example, to track them across the sky. If you have ever seen a TV satellite dish, you know these do not move but are fixed to a house or a pole, yet they are aimed at the satellite at all times. How is this possible? At what distance from (the center of) Earth do these geosynchronous satellites orbit? As viewed in NC, at what altitude above the horizon do they appear?
3. Weight loss plans:

(a) When you stand on a scale what the scale registers is the force with which it has to push you up to oppose the Earth’s gravitational attraction pulling you down. To compute this force, use Newton’s result that from the outside, the gravitational attraction of a spherical homogeneous mass can be computed by replacing the entire Earth with a point-like object, its mass equal to the Earth’s mass (which you computed in the previous problem set) located at the center of the Earth, a distance of 6378 km from you standing on the surface. If my mass is 60 kg, compute my weight. Note: you should be using Newton’s Universal equation as we wrote it in class. The result should agree with a more familiar expression.

(b) *Plan 1: Move to the Equator.* As briefly mentioned in class, the above calculation is strictly valid only at the Earth’s poles. Away from the poles, the scale need not cancel gravity completely. Some part of my weight is required to accelerate me towards the center of the Earth as I move along a circular path around it with the spinning Earth. So, if I am on the equator, a scale should register a smaller weight than it would at the poles. Compute the difference.

(c) In fact, things are even better than this. Because it has been spinning for a long time, the effect you just computed has caused the surface of Earth to become slightly *oblate,* or deviate from a spherical shape in the direction of the shape of an M&M. Of course, the Earth is much closer to a sphere than a candy. But at the equator you are indeed farther from the center of Earth than at the pole, by about 0.34%. This contributes to the success of this weight-loss strategy, because the gravitational attraction decreases with increasing distance. Compute the contribution of this effect to my weight loss.

(d) *Plan 2: Stand under the Moon.* Another strategy for losing weight uses the gravitational pull of the Moon. When I stand directly underneath our companion, its gravitational attraction opposes that of the Earth. The Moon’s mass is $7.35 \times 10^{22}$ kg. Compute the gravitational attraction the Moon applies to me. If Earth disappeared, with what acceleration would I fall towards the Moon? Compare this with the Moon’s acceleration towards Earth which you computed in problem 3. Can you explain the ratio of the two?
(e) Well, like most weight loss promoters, I fudged. The Moon’s gravitational pull acts on the Earth as well as on me, and so the entire Earth is accelerating towards the Moon with the acceleration you calculated in (c)! Thus the scale must push on me to accelerate me towards the Moon, in addition to opposing Earth’s gravity. Is my plan ruined? Not completely. You see, the Earth accelerates towards the Moon with an acceleration determined by the average of the Moon’s gravitational attraction on various parts of Earth; equivalently, this is the acceleration corresponding to the distance from the Moon to the Earth’s center. But standing under the Moon I am about 6378 km closer to the Moon than the Earth’s center, and since gravitational forces decrease with distance, the force on me here is stronger than it would be at the distance of Earth’s center. Thus there is a “leftover” or tidal force; if the scale cancelled the Earth’s pull on me precisely, the Moon’s attraction would accelerate me up slightly more than the Earth is accelerating and I would lift off. This does not happen, because the scale pushes me up just enough to keep me on Earth, slightly less than my weight. Compute the difference, which comprises the net weight loss using this strategy.

4. The kinetic energy of an object of mass \( m \) moving at a speed \( v \) is given by \( E_K = \frac{mv^2}{2} \); the gravitational potential energy of a system of two objects of mass \( M, m \) separated by a distance \( R \) is \( E_P = -\frac{GMm}{R} \). Note that the objects in which we are interested are usually either spherical, so that \( R \) may be taken as the distance between centers, or so small relative to \( R \) that we may safely use the distance to any point on them.

(a) Imagine a spherical asteroid of radius 2 km. and density \( 2500 \text{ kg/m}^3 \) hitting the Earth with a velocity of \( 25,000 \text{ m/sec} \). What is the kinetic energy carried by the asteroid (in Joules)? In the collision this would mostly be converted to heat in an immense fireball and shockwave. How does this compare to the energy released by the bomb that destroyed Hiroshima, which had a “yield” of 20 kilotons? The yield of nuclear weapons is given in terms of the mass of TNT that would release the same amount of energy when it explodes. A kiloton (1000 tons) of TNT releases \( 4.2 \times 10^{12} \text{ J} \) of energy.

(b) As the asteroid crashes towards Earth, it is accelerated by Earth’s gravity. If the asteroid were dropped from rest at an altitude (above Earth’s surface) of 200 km, we can describe its acceleration towards Earth as a conversion of potential to kinetic energy. Compute the difference between the asteroid’s potential energy
on the Earth’s surface and at the initial altitude, note that the former is smaller (more negative). This difference, according to the conservation of energy, is what will be converted to kinetic energy in the fall. How does this compare to the result in (a)?

(c) Imagine an asteroid that starts at rest in the asteroid belt, a distance of, say, 3 AU from the Sun and 2 AU from Earth. As it falls towards Earth, it is accelerated by both the gravitational attraction of the Sun and that of the Earth. Neglecting the motion of the Earth, find the potential energy the asteroid loses as it falls to Earth’s surface as a sum of two terms, one corresponding to each of the two forces above. Which force is responsible for the result in (a)?

(d) Note that as an object retreats farther and farther from Earth the potential energy associated to Earth’s gravitational field becomes closer and closer to zero (approaching this from below). This means if we launch an object off the Earth with an initial total energy of zero, then as it rises against the Earth’s gravitational attraction this object will slow but continue to rise, coming to a stop when it is “infinitely distant” from Earth (in practice, the object will have freed itself from Earth’s gravity only to find itself in orbit about the Sun, as we saw above). Find the speed with which an object must move at Earth’s surface so as to have zero total energy. This is called the escape velocity from Earth.