Introduction to Astrophysics

Unit 2:
Newton’s Universe
Signs of the Times

• Astronomy and timekeeping are always closely related – we want our time to match what happens.
• Our 24-hour **days** are adjusted to **mean solar day**.
• Our months are approximately **lunar**.
• Our **years** match orbit – **365.2564** days is a **sidereal orbit**.
• **Tropical orbit** is **365.2422** days (precession).
• **Julius Caesar** got 365.25 so invented **leap years**.
• **Pope Gregory XIII** (1582) corrected for the **.0078**
So Far

• Our cosmos now has **moving parts**
  – **Sun** moves around Celestial Sphere to the **East**, completes one revolution in a **year**. The **ecliptic** tilted relative to **celestial equator** by 23.5° about **equinoxes** and precesses **West** every 26,000 years
  – **Moon** moves around Celestial Sphere to the **East**, completes one revolution in a **month**. Moon’s **orbit** tilted relative to **ecliptic** by 5° about **line of nodes** and precesses **West** every 18.6 years

• The model now explains day/night, lunar phases, eclipses
• What else moves?
Wanderers

• Five wandering stars (planets) also move along paths very near the ecliptic
• Rates of motion vary among planets, so each located on its own sphere. We now have seven spheres
• Planetary motion less regular than Sun or Moon. Rate changes and sometimes turns retrograde
Ptolemaic Astronomy

• Hipparchus (150BC): Planets move on epicycles which move along deferents
• Ptolemy (150AD) elaborates model to account for small deviations
• Ptolemaic model predicts planetary motions accurately – successful scientific model
• Ptolemaic order of spheres: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn; Fixed Stars
How it works

• **Venus** never too far from Sun
• Its **deferent** circles about once a **year**
• **Epicycle** rotation accounts for periodic change in **elongation**
• When **West/East** of Sun we see **morning/evening star**
Antikythera

- 1901 pearl fishers find sunk ship from 80BC
- Many bronze artifacts including *astronomical calculator*
- Predicts positions of planets, moon phases, eclipses using Ptolemaic model
An Alternate Model

• **Heliocentric model (Aristarchus 270BC):** planets – and Earth - orbit the Sun

• Perceived motion of planets results from relative motion of planet and Earth

• Retrograde motion explained simply

• **Copernicus (1543) produces a detailed heliocentric model competitive with Ptolemy**
Periods

• Planet orbits Sun in time $P$ – sidereal period
• Earth orbits Sun in time $E$ - sidereal year
• Planet’s synodic period $S$ is time between conjunctions/oppositions for inferior/superior planets
• Can relate them mathematically
Computing Periods

\[ \frac{S}{P_1} = \frac{S}{P_2} + 1 \]

\[ \frac{1}{P_1} = \frac{1}{P_2} + \frac{1}{S} \]
So Which is it?

• Both models produce predictions matching observations of apparent motion
• Culturally difference is huge: does Earth move? Is it central?
• Scientifically difference is in motion of planets in 3d.
• Other things change in the heavens: Comets come and go and their motion does not fit well with either model
• Real determination: better observations with new technology
• Leads to new understanding of fundamental laws that are universal. That is the scientific revolution
Data Gathering

- Tycho Brahe (1580) makes observations of planetary motion with improved accuracy.
- J. Kepler (1609) finds that these are consistent with a heliocentric model governed by three laws.
Kepler’s Laws

• 1. Orbit of a planet is an **ellipse** with **Sun** at one focus
• What’s at other focus? Nothing, not even the same for all planets
• Eccentricity small: 0.017 for Earth, 0.2 for Mercury.
• 2. Line from Sun to planet sweeps out equal **areas** in equal **times**
• Planet moves **faster** near **perihelion**, slower near **aphelion**
• **Comets** in highly eccentric orbits – dramatic effect
Kepler’s Third

• 3. Square of sidereal period proportional to cube of semimajor axis

\[ P^2 = K a^3 \]

Same \( K \) for all planets!
Galileo’s Smoking Scope

• New technology: Galileo (1610) turns telescope up

• Finds
  – phases of Venus showing it orbits Sun
  – Galilean moons orbiting Jupiter
  – Mountains on Moon, spots on Sun, ears on Saturn
This is Progress

• Galileo studies heavens as a physical system to be observed
• Kepler’s laws predict planetary motion with unprecedented accuracy from simple model
• They are universal. In fact, they govern orbiting systems from Solar System to Saturn’s Moons to...electrons in an Atom (with different $K$)
• Such universality is a hint of underlying fundamental laws
• Galileo also studied mechanics (science of motion) and formulated principle of inertia: An object will retain its state of motion unless disturbed externally
• It took Newton and new math to find them
Motion

• State of motion is velocity $\vec{v}$ - speed and direction in $\frac{m}{s}$
• Rate of change of $\vec{v}$ is acceleration $\vec{a}$ in $\frac{m}{s^2}$
• Acceleration can be speeding, slowing, or turning and is directed in direction of change
Circular motion

• We found that \( \vec{a} \) directed to center and of constant magnitude. If radius is \( R \) and speed \( v \), what is magnitude of \( a \)?

\[
a = \frac{v^2}{R}
\]
Mechanics

- **Acceleration** due to a force applied by another object: \( \vec{F} = m\vec{a} \)

- \( m \) is a property of **object** mass - in **kg**

- \( \vec{F} \) is measured – in \( \text{N} = \text{kg m/s}^2 \)

- When object **A** applies a force \( \vec{F} \) to **B**, then **B** applies a force \( -\vec{F} \) to **A**
Weight and Mass

• Weight of an object is the force gravity applies to it
• We know objects fall with acceleration \( g = 9.81 \text{ m/s}^2 \) so force of gravity is \( F = mg \)
• \( g \) is property of Earth
• My mass is 59 kg. My weight on Earth is 579 N
Conservation Laws

• Mathematical theorems follow from Newton
• Momentum $\vec{p} = m\vec{v}$ then $\vec{F}$ is rate of change of $\vec{p}$
• So if $A$ and $B$ act on each other, $\vec{p}_A + \vec{p}_B$ does not change – they exchange momentum but can’t create or destroy it. Momentum conserved
More Conservation

• With a little more math, see that a circular version – angular momentum $L = mvR$ is also conserved

• This will be incredibly important to us – things in space spin
Energy

• If gravity is the only force acting on an object, can show that total energy is constant

\[ E = \frac{mv^2}{2} + mgh \]

• In general, other forces act. Find that this introduces more kinds of energy: sound, light, heat, chemical, electric, nuclear, etc. but total is conserved

• Units of energy: \[ J = \frac{\text{kg m}^2}{\text{s}^2} \]
This is Everything

• $\vec{F} = m\vec{a}$. The rest is details

• If we can figure out forces this is a way to predict from where things are today where they will be in future (or were in past):

positions, velocities \rightarrow forces \rightarrow accelerations

advance t
Attractive Logic

• If an object of mass \( m \) moves in a circle of radius \( R \) with uniform speed \( v \) there must be a force acting, directed to center, of magnitude \( F = \frac{mv^2}{R} \)

• Moon orbits Earth so force directed towards Earth

• We notice Earth applies such a force to apples. Could this be the same force?

• Planets orbit Sun in (almost) circular orbits at (almost) uniform speed. Does Sun apply the force this implies on all planets?

• If so, Earth must apply a force to Sun directed towards Earth

• It all hangs together!!
Kepler and Newton

- **Planet** of mass $m$ orbits **Sun** at radius $R$ with speed $v$
- **Force** Sun applies to planet is thus $F = \frac{mv^2}{R}$
- **Kepler says**
  - $v^2 = \left(\frac{2\pi R}{P}\right)^2 = \frac{4\pi^2}{R} \frac{R^3}{P^2} = \frac{4\pi^2}{K_\odot R}$
- **Find**
  - $F = \frac{4\pi^2 m}{K_\odot R^2}$
It’s Universal

• **Sun** applies a force given by \( F = \frac{4\pi^2 m}{K_\odot R^2} \) to each planet, with same \( K_\odot \)

• So each planet applies force of same magnitude to **Sun**

• Law does not single out planet from Sun, so must have \( F = \frac{GM_\odot m}{R^2} \) so \( K_\odot = \frac{4\pi^2}{GM_\odot} \)
Really Universal

- Earth also attracts the Moon, and all objects near it.
- So Moon and all other objects attract Earth.
- Everything attracts everything else!

\[ F = \frac{G m_1 m_2}{R^2} \]

- Measured Newton’s constant:
  \[ G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \]
Thinking about Orbits

• Why doesn’t the Moon fall on Earth?
• It does! Moon is constantly accelerating towards Earth. Orbiting is falling without ever hitting the ground
Success!

- Newton’s equations predict Kepler’s laws.
- Also show there are other types of orbits.
- Neptune discovered mathematically in details of Uranus’s orbital motion.
Even More Generally

- Newton’s law is **Universal**. Apply to any two objects orbiting under mutual gravity. Find elliptical orbit about center of mass with

\[ P^2 = K \alpha^3 \]

\[ K = \frac{4\pi^2}{GM_{\text{total}}} \]
Example: Low-Earth Orbit

- ISS orbits at an altitude $h = 370 \text{ km}$ so has orbital radius $R = 6371 + 370 = 6741 \text{ km}$

- Period given by

$$P = 2\pi \left( \frac{R^3}{GM_\oplus} \right)^{1/2}$$

$$= 2\pi \left( \frac{(6.741 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.972 \times 10^{24}} \right)^{1/2}$$

$$= 5510 \text{ s} = 91.8 \text{ m}$$
Working with Newton

• When I am near Earth, every bit of Earth exerts a bit of attractive force, directed towards it. To get total, add them up.

• Newton shows that for any round shell the total force it exerts is zero if you are inside, but same as if entire mass were in the center if you are outside

• Adding it all up, outside Earth we can compute force by considering entire mass located at center
Ronen’s Gravitas

- Force on me \( m_R = 59 \text{ kg} \)

\[
F = \frac{GM_\oplus m_R}{R_\oplus^2} = \frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{(6.371 \times 10^6)^2} m_R = 9.82m_R = 579 \text{ N}
\]
Gravity Here and There

- As I get further from Earth force decreases

\[ F = \frac{GM_\oplus m_R}{(R_\oplus + h)^2} \]
\[ = m_R g \left( \frac{R_\oplus}{R_\oplus + h} \right)^2 \]
\[ = m_R g \left( 1 + \frac{h}{R_\oplus} \right)^{-2} \]
\[ F \sim m_R g \left( 1 - 2 \frac{h}{R_\oplus} \right) \quad h \ll R_\oplus \]

- Newton:

\[ (1 + x)^a \sim 1 + ax \quad x \ll 1 \]
Potential Energy

- We said potential energy was $mgh$. This true if force constant so valid near surface.
- Since force decreases, height gain costs less energy at large distance. Find
  \[ U = -\frac{GM\oplus m}{R} \]
Energy in Orbit

• At radius $R$ potential energy $U = -\frac{GMm}{R}$
• Speed $v^2 = \frac{4\pi^2}{KR} = \frac{GM}{R}$ so kinetic energy $\frac{mv^2}{2} = \frac{GMm}{2R}$
• Total energy $E = \frac{mv^2}{2} + U = -\frac{GMm}{2R}$
• Negative energy orbits are bound, closed
• Positive energy orbits unbound
The Principle of Equivalence

• S. Hawking is **weightless** because gravity is weaker in space?

• No! $h=400km$ so

\[
F \sim mg \left(1 - 2\left(\frac{h}{R_\oplus}\right)\right)
\]
\[
= mg \left(1 - 2\left(\frac{400}{6371}\right)\right) = 0.87mg
\]

• Hawking is in **free fall**

In free fall there is **no gravity**
Leftovers

• **Earth** is in free fall under gravity of Sun, so
• Sun’s gravity has no effect on Earth!
• **Almost** none. There are remnants of gravity even in freefall: **tidal forces**
• These are due to the fact that gravitational **acceleration** is different at different points. So not all points of an extended object can possibly be simultaneously in free-fall
• **Difference** $a_T$ in free-fall acceleration (from center of Earth) acts as a tidal “force” $F_T = ma_T$
\[ a_+ = \frac{GM_\odot}{D_\odot^2} \]

\[ a_+ = \frac{GM_\odot}{(D_\odot - R_\oplus)^2} \]

\[ a_- = \frac{GM_\odot}{(D_\odot + R_\oplus)^2} \]

\[ a_+ \sim a_\oplus \left(1 + 2\frac{R_\oplus}{D}\right) \]

\[ a_- \sim a_\oplus \left(1 - 2\frac{R_\oplus}{D}\right) \]
How Strong is this Force?

\[ a_T^\odot = \frac{2G M_\odot R_\oplus}{D_\oplus^3} \]

\[ = 2 \frac{G M_\oplus}{R^2_\oplus} \left( \frac{M_\odot}{M_\oplus} \right) \left( \frac{R_\oplus}{D_\odot} \right)^3 \]

\[ = 2g \left( \frac{1.989 \times 10^{30}}{5.972 \times 10^{24}} \right) \left( \frac{6371}{1.496 \times 10^8} \right)^3 \]

\[ = 5.14 \times 10^{-8} g \]
What about the Moon?

\[ a_T^{\text{Moon}} = a_T \left( \frac{M_{\text{Moon}}}{M_\odot} \right) \left( \frac{D_{\text{Moon}}}{D_\odot} \right)^{-3} \]

\[ = a_T \left( \frac{7.348 \times 10^{22}}{1.989 \times 10^{30}} \right) \left( \frac{3.844 \times 10^5}{1.496 \times 10^8} \right)^{-3} \]

\[ = 2.2 a_T \]
The Tides

• Moon deforms water so bulge faces Moon. As Earth rotates, bulge moves around Earth so tides repeat every 24h 48m
• Earth’s rotation drags bulge East so lags Moon by about 12m
• Sun exerts tidal force towards Sun about ½ as strong. At full/new Moon act together creating intense spring tides. At quarter Moon counteract to create weak neap tides
Even More Tides

• When Moon formed – molten and closer - Earth’s tidal forces deformed it so it froze with permanent bulge. Tidal forces keep this bulge aligned with direction to Earth: tidal locking is why we always see same side of the Moon.

• Since tidal bulge on Earth is dragged East of Moon, tidal force of Moon tries to align it. This in fact slows Earth’s rotation, transferring angular momentum to the Moon which thus recedes into higher orbit (G. Darwin, 1898)
What Now, Aristotle?

• Applying universal laws leads to unified understanding of many phenomena!

• In space, everything is in free-fall. Trajectories are Keplerian orbits. Internal structure controlled by tidal forces

• $\vec{F} = m\vec{a}$ is powerful. Learn more about matter and forces