Introduction to Astrophysics

Unit 5:
Stars
Plan

• Stars are **Suns** so start by learning what we can about our local star

• To compare to other stars, need to find **luminosity, temperature, size,** and **mass**

• Combine **statistics** with stellar **models** to understand how stars work
The Sun Shines – but How?

• Sun is **big** and **hot** so **luminous** \[ L_\odot = 3.83 \times 10^{26} \text{ W} \]
• How does it **stay** hot? \[ M = 1.989 \times 10^{30} \text{ kg} \]
• **Chemical** (rearrange **electrons** - electromagnetic) burning produces \[ 10^{-19} \text{ J} \] per atom, or \[ 6 \times 10^7 \text{ J} \] per kg.
• Need to burn \[ 6.4 \times 10^{18} \text{ kg/s} \] so run out in \[ 10^4 \text{ y} \]
• **Kelvin-Helmholtz** (gravitational) energy would last \[ 10^7 \text{ y} \]
Nuclear Physics

• Why don’t nuclei break up under electric repulsion?
• A strong attractive force binds nucleons
• Short-range $\sim 10^{-15}\text{ m}$ since atoms do not collapse
Nuclear Energy

- Rearranging nucleons recover nuclear energy
- In large nuclei distant nucleons barely attract
- Breaking up – fission – or α emission recover electromagnetic energy
- Heats planets powers reactors
Fusion?

- In small nuclei, less attractive interactions
- Liberate nuclear energy by fusion to Helium
- Problem: Hydrogen is all protons
- Strong interactions cannot change a proton to a neutron
Weak Interactions

- **Something** can do this!
- And the **inverse**
- A free **neutron** decays in **15min**
  \[ n \rightarrow p^+ + e^- + \bar{\nu}_e \]
- **Weak nuclear force** mediates this decay
Some Questions and Answers

• Can a force change one particle into another? Yes
• Is a neutron just a tiny Hydrogen atom? No
• What is $\overline{\nu}_e$?
• Are there any rules?

• Conservation Laws
  – Mass-Energy $E = mc^2$
  – Momentum
  – Angular Momentum
  – Electric Charge
  – Electron Number

• Weak interaction: rare
Particle Physics

- **Antiparticle**: same mass, opposite charges
- Neutrinos almost massless, weakly interacting
- Discovered as missing energy in $n$ decay

<table>
<thead>
<tr>
<th>Particle</th>
<th>Q</th>
<th>$N_e$</th>
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<td>0</td>
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<tr>
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<tr>
<td>$\gamma$</td>
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</table>
Solar Energy

- \( p-p \) chain is source of

\[
\begin{align*}
p^+ + p^+ & \rightarrow d^+ + e^+ + \nu_e \\
d^+ + p^+ & \rightarrow ^3\text{He}^{2+} \\
^3\text{He}^{2+} + ^3\text{He}^{2+} & \rightarrow ^4\text{He}^{2+} + p^+ + p^+
\end{align*}
\]

\[4p^+ \rightarrow \alpha^{2+} + 2e^+ + 2\nu_e + 4.3 \times 10^{-12} \text{ J}\]

- Sun could last \( 10^{11} \text{ y} \)
What it Takes

• To initiate fusion, protons must overcome electric repulsion
• One proton must inverse $\beta$ decay before highly unstable $^2\text{He}$ breaks up
• Requires temperatures of $10^6\text{ K}$ - only in core
• Inefficient because weak process required
How Do We Know?

- **Theory** *(Eddington, Bethe 1932)* first
- **Davis, Bahcall (1968):** Detect the $\nu_e$
- Pro: Penetrate Sun
- Con: Penetrate detector
- Flux at Earth: $10^{11}$ $\frac{\nu_e}{\text{m}^2 \text{s}}$

- Put a tank with $380 \text{ m}^3$ of Chlorine in Homestake Gold Mine
  \[ {^{37}\text{Cl}} + \nu_e \rightarrow {^{37}\text{Ar}} + e \]
- Requires high-energy $\nu_e$ produced in other processes
- Expect one atom per six days
Where Are the Neutrinos?

- Flux Found is less than predictions
- Is Solar Model wrong?
- Is detector model wrong?
- Decided in 2001 by SNO: particle physics
## More Particles, More Charges

<table>
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<tr>
<th>Particle</th>
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<th>$N_\mu$</th>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>?</td>
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</table>
So What?

• Neutrinos change spontaneously en route
• $pp$ process produces $\nu_e$
• When they arrive, $1/3$ are $\nu_e$
• This implies, in particular, that neutrinos are not massless although light.
Studying the Sun

- Solar models together with helioseismology provide interior structure between core and photosphere
- Density, pressure, temperature increase with depth for hydrostatic equilibrium
Solar Structure - Core

- **Core:** \( R \leq 0.25R_\odot \)
  \[
  1.57 \times 10^7 \text{ K} \geq T \geq 7 \times 10^6 \text{ K} \\
  1.5 \times 10^5 \text{ kg/m}^3 \geq \rho \geq 2 \times 10^4 \text{ kg/m}^3 \\
  M \sim 0.4M_\odot
  \]

- **Stable equilibrium:** fusion rate decreases/increases: core contracts/expands increasing/decreasing rate

- **Luminosity** determined by mass
Solar Structure – Inner Mantle

- **Radiation Zone:**
  
  \[0.25R_\odot \leq R \leq 0.7R_\odot\]
  
  \[7 \times 10^6 \text{ K} \geq T \geq 2 \times 10^6 \text{ K}\]
  
  \[2 \times 10^4 \text{ kg/m}^3 \geq \rho \geq 10^3 \text{ kg/m}^3\]

- **Heat transfer:** Radiation diffusion in charged plasma

- **Transit time:** \(1.7 \times 10^5 \text{ y}\)
Solar Structure – Outer Mantle

- **Convection Zone:**
  
  \[0.7 R_\odot \leq R \leq R_\odot\]
  
  \[2 \times 10^6 \text{ K} \geq T \geq 5780 \text{ K}\]
  
  \[10^3 \text{ kg/m}^3 \geq \rho \geq 2 \times 10^{-4} \text{ kg/m}^3\]

- **Heat Transfer:** Convection produces granular structure of photosphere
Solar Atmosphere

- Sun extends beyond photosphere
- Density low but temperature increases with altitude
- Chromosphere:
  \[
  h \leq 2000 \text{ km} \\
  5780 \text{ K} \leq T \leq 50,000 \text{ K} \\
  2 \times 10^{-4} \text{ kg/m}^3 \geq \rho \geq 10^{-10} \text{ kg/m}^3
  \]
  Observe by $H_\alpha$ line
Corona

- **Corona:**
  - $2000 \text{ km} \leq h \leq 1.3 R_\odot$
  - $T \sim 2 \times 10^6 \text{ K}$
  - $\rho \sim 3 \times 10^{-12} \text{ kg/m}^3$
- Visible during **Eclipse** or with **coronagraph**
- Observed in **UV, X-Ray**
- High temperature allows escape: **Solar wind**
Blemishes

- First recorded Gan De 364BC
- Galileo used them to find rotation period 25.4d
- Dark because cooler 4000K
- Wilson 1769: depressions in photosphere
Patterns

- Sunspot number varies in 11y cycle
- Sunspot pairs appear first at mid-latitudes and later near equator
- Spots are regions of increased magnetic field choking convection
- Pair polarity consistent in hemisphere/cycle reverses between cycles
Solar Magnetism

- Solar field not simple dipole
- Differential rotation of charged plasma deforms field in convection zone
- High fields at surface produce reconnection events
- Reconnection releases energy in field, reverses polarity every 11y
Magnetic Storms

• Reconnection releases magnetic energy accelerating charged particles
• Sudden release of up to $6 \times 10^{25}$ J in flare heats gas to $10^7$ K
• More violent: Prominences, Coronal Mass Ejections
• Cause geomagnetic storms
Stars in 3d - Parallax

• To measure distance to star – measure change in its apparent position as seen from different points.

• Measure this relative to more distant stars

• Most distant observatories: same place, different seasons

\[
\frac{D}{1 \text{ AU}} = \frac{206265''}{p}
\]

\[
\frac{D}{1 \text{ pc}} = \frac{1''}{p}
\]
First Steps on the Ladder

• First stellar parallax measured by Bessel 1838
• Shatters celestial sphere extends 3d to stars
• What’s an AU? Best measurement: radar telemetry to planets. This determines pc
• Hipparchos 1989 measures 120,000 stars, leading to current catalog of 2,500,000
• Gaia 2013 will vastly extend this
Proper Motion

- Some nearby stars observed to move relative to distant stars
- Find tangential velocity $v_T$ from angular proper motion $\mu$
  \[ v_T = 4.74\mu D \]
- Radial velocity from Doppler
  \[ v_r = c\left(\frac{\lambda}{\lambda_0} - 1\right) \]
Big Dipper?
If We Know the Distance

- Can measure **brightness** and compute luminosity
  \[ L = 4\pi D^2 b \quad \frac{L}{L_\odot} = \frac{b}{b_\odot} \left( \frac{D}{1\text{ AU}} \right)^2 \]
- Measure **color** (spectrum) to find **temperature**
  \[ T = \frac{0.0029 \text{ m}}{\lambda_{\text{max}}} \text{ K} \]
- Compare the two to find **radius**
  \[ R = \left( \frac{L}{4\pi\sigma T^4} \right)^{1/2} \quad \frac{R}{R_\odot} = \left( \frac{L}{L_\odot} \right)^{1/2} \left( \frac{T}{T_\odot} \right)^{-2} \]
A Better Thermometer

- Blackbody spectrum too broad and subject to distortion by medium
- Stellar line spectra give better data
- Atmosphere composition and ionization state indicate temperature
Spectral Lines and Temps
<table>
<thead>
<tr>
<th>Type</th>
<th>Color</th>
<th>Temperature</th>
<th>Lines</th>
<th>Prevalence</th>
<th>Examples</th>
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<td>O</td>
<td>Blue</td>
<td>&gt; 33,000</td>
<td>He(^0), He(^+), weak H</td>
<td>&lt;0.00003%</td>
<td>Orion’s Belt</td>
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<tr>
<td>B</td>
<td>Blue-White</td>
<td>10,000-33,000</td>
<td>He(^0), strong H</td>
<td>.13%</td>
<td>Spica, Rigel</td>
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<tr>
<td>A</td>
<td>White to Blue-White</td>
<td>7500-10,000</td>
<td>No He, Very strong H, some metal ions</td>
<td>.6%</td>
<td>Sirius, Vega</td>
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<tr>
<td>F</td>
<td>White</td>
<td>6000-7500</td>
<td>strong H, many metal ions</td>
<td>3%</td>
<td>Procyon, Polaris</td>
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<tr>
<td>G</td>
<td>Yellowish White</td>
<td>5200-6000</td>
<td>Weak H, many metals</td>
<td>7.6%</td>
<td>Sun, Capella</td>
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<tr>
<td>K</td>
<td>Orange</td>
<td>3700-5200</td>
<td>Neutral metals</td>
<td>12.1%</td>
<td>Arcturus, Aldebaran</td>
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<tr>
<td>M</td>
<td>Red</td>
<td>2000-3700</td>
<td>Neutral Metals, molecular bands</td>
<td>76.5%</td>
<td>Betelgeuse</td>
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</table>
Stellar Statistics

- We can now use all this to study properties of stars as a population
- Luminosity varies hugely
- Radius varies less
- Patterns appear when you arrange it right: Herzsprung-Russell 1910
Luminosity Classes

• How to tell an orange giant from a much smaller orange MS star?
• Morgan, Keenan, Kellerman 1943: Spectral lines for small stars show effects of higher pressure, density
• Sun’s spectral class is G2V
Spectroscopic “Parallax”

- From spectrum can obtain spectral class
- Extract luminosity from H-R diagram
- Use brightness to find distance
- No parallax

\[
\frac{D}{1\text{AU}} = \sqrt{\frac{L}{L_\odot} \frac{b_\odot}{b}}
\]
An Example: Alphecca

- Dim **white** star. Flux \( b_{\alpha CB} = 2.63 \times 10^{-12} b_\odot \)
- Spectrum classifies it as **A0V** \( T_{\alpha CB} = 9700 K \)
- Luminosity from HR \( L_{\alpha CB} = 74 L_\odot \)
- Radius:

\[
R_{\alpha CB} = \left( \frac{L_{\alpha CB}}{L_\odot} \right)^{1/2} \left( \frac{T_{\alpha CB}}{T_\odot} \right)^{-2} \\
R_\odot = \sqrt{74} \times \left( \frac{9700}{5780} \right)^{-2} = 3R_\odot
\]
Alphecca

- **Distance:**
  \[ D_{\alpha CB} = \sqrt{\frac{L_{\alpha CB}}{L_{\odot}}} \frac{b_{\odot}}{b_{\alpha CB}} \text{ AU} = \sqrt{\frac{74}{2.63 \times 10^{-12}}} \]
  \[ = 5.30 \times 10^6 \text{ AU} = 25.7 \text{ pc} \]

- **Hipparchos:**
  \[ p_{\alpha CB} = 0.04365'' \]
  \[ D_{\alpha CB} = 22.9 \text{ pc} \]

- 10% error typical
Partners

• About 1/5 of all stars are gravitationally bound to a partner
• Glossary:
  – Visual: can see both
  – Optical Double: not binary
  – Non-visual: other methods
• $\alpha$-Centauri AB is a triple with Proxima Centauri
α-Centauri is Famous

• As nearest star to Earth this has been of interest to many

• October, 2012: an Earth-sized planet detected orbiting α-Centauri B at 0.04 AU
With Benefits

• Star position gives a projection of orbit on tangent plane
• Can find radial component by Doppler shift
• Get complete orbit

• Given period and radius find mass

\[
\frac{M_1 + M_2}{M_\odot} = \left(\frac{R}{1\text{AU}}\right)^3 \left(\frac{P}{1\text{y}}\right)^{-2}
\]

• Plotting motion of both partners find \( R_1, R_2 \)

\[
M_1 R_1 = M_2 R_2
\]
Spectroscopic Binaries

• If stars too close to resolve can distinguish periodic Doppler shift: Spectroscopic Binary
• If we can see both stars: double-line binary
• Often see only one
Learning from Spectrum

• **Measure:** \( v_1, v_2, P \)  \( v = c (\lambda / \lambda_0 - 1) \)

\[
R_{1,2} = \frac{v_{1,2}P}{2\pi} \quad M_1 R_1 = M_2 R_2 \quad M_1 v_1 = M_2 v_2
\]

\[
M = M_1 + M_2 = M_1 (1 + v_1/v_2) = M_1 (v_2 + v_1)v_2
\]

\[
R = R_1 + R_2 = \frac{P v_1}{2\pi} + \frac{P v_1}{2\pi} = (v_1 + v_2) \frac{P}{2\pi}
\]
Finding Masses

\[ P^2 = \frac{4\pi^2}{GM} R^3 \quad R = \frac{P}{2\pi} (v_1 + v_2) \quad M = (v_1 + v_2)M_1/v_2 \]

\[ M = \frac{4\pi^2 R^3}{GP^2} = \frac{4\pi^2 P^3}{(2\pi)^3 GP^2} (v_1 + v_2)^3 \quad M_1 = \frac{v_2 P}{2\pi G} (v_1 + v_2)^2 \]

• We can find masses!
• Caveat: Orbit may tilt, we measure only \( v_r \)
• If we only see one star only know \( v_1 \)
Eclipsing Binaries

- As with planets, binaries in which one star transits the other provide more information.
- Light curve tells us about period, sizes, and temperatures.

Observing Doppler shifts and light curve get more complete data.
Alphecca

- Alphecca is a double-line **eclipsing** binary with
  \[ P = 17.36 \, \text{d} \]
  \[ v_2 = 9.9 \times 10^4 \, \text{m/s} \]
  \[ v_1 = 3.5 \times 10^4 \, \text{m/s} \]

- **Doppler** measurements yield

- **Predict:**
  \[ M_1 = \frac{P}{1 \, \text{y}} \frac{v_2(v_1 + v_2)^2}{(29.78 \, \text{km/s})^3} M_\odot = \frac{17.36}{365.25} \frac{99 \cdot 134^2}{29.78^3} = 3.2 M_\odot \]
  \[ M_2 = \frac{v_1}{v_2} M_1 = 1.1 M_\odot \]
How Did We Do?

- Spectrum agrees: αCB B is G5V
- Light Curve data refine this (eccentricity 0.37)
- Best Fit:

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<td>$M/M_\odot$</td>
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<td>0.92</td>
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<tr>
<td>$R/R_\odot$</td>
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<tr>
<td>$T(K)$</td>
<td>9700</td>
<td>5800</td>
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<tr>
<td>$L/L_\odot$</td>
<td>74</td>
<td>0.81</td>
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</table>
Mass Statistics

- Add mass for **main sequence** to our plot
- Masses vary **little**
- Model: Stars are the **same**: mass determines rest
- Heavy stars **hot, luminous**
Mass-Luminosity Relation

- Find approximately
  \[ \left( \frac{L}{L_\odot} \right) = \left( \frac{M}{M_\odot} \right)^{3.5} \]

- Borne out by models: Mass compresses star increasing rate of fusion

- If amount of Hydrogen available for fusion is near constant fraction, big stars run out sooner

- OB stars are young!
Basics

• Stellar **modeling** matched to **data** tells us about how stars work
• Main-Sequence stars fuse Hydrogen to Helium in core
• Hydrostatic **Equilibrium** determines rate of fusion and density profile from **mass**
CNO Chain

- In large stars $M > 1.5M_\odot$ core hot and CNO chain dominates fusion
- Rate rises rapidly with temperature
Size Matters

- Mechanisms of heat transfer depend on mass
- In small stars, entire volume convective so all available to fuse in core
- In large stars, radiation and convection zones inverted
Expansion by Contraction

- As a main sequence star ages core enriched in Helium
- Rate of fusion decreases – temperature and radiation pressure decrease
- Number of particles decreases – thermodynamic pressure decreases
- Core contracts and heats
- Fusing region grows
- Luminosity increases
- Envelope expands

- Sun now 25% brighter than when it formed
- Core now 60% Helium
- Continues to brighten – heating Earth
- In 1-3Gy could be uninhabitable?
- Orbit stable out to 1Gy?
Summary

• For 90% of stars we have a good understanding of how they work
• This comes from careful observation and detailed modeling
• Where do the rest come from?
• What happens when core is all Helium??